Prolate Spheroidal Wave Functions Of Order Zero Mathematical Tools For Bandlimited Approximation Applied Mathematical Sciences

Growing request for wideband communications requires innovation in power efficiency and signal processing. Without the use of any peak to average power ratio (PAPR) reduction technique, the efficiency of power consumption at the transmitter end becomes very poor. PAPR reduction in this work is accomplished based on using a unique class of functions, prolate spheroidal wave functions (PSWFs). The difficulty arises from the fact that these pulses do not belong to the Nyquist family. A zero forcing equalizer (ZFE) is designed to compensate intersymbol interference (ISI), and its performance is studied under the presence of AWGN. Considering PAPR and ISI as the constraints of communication systems, based on the properties of PSWF, a set of pulses with minimum ISI with respect to a specific amount of PAPR is achieved by defining an optimization problem. The desired level of PAPR is considered to be moved to the constraint set to convert the multi-objective problem into a single objective problem. The results of the numerical optimization of both ISI and PAPR are presented along with a couple of examples of comparison between the resultant pulse and the conventional square root raised cosine. It is shown that by achieving the same level of PAPR of the SRRC, the obtained pulse is a close approximation of SRRC. An implementation based on state variable filters is introduced to realize PSWF for high speed applications. An example based on this approach is presented to compare the finite pole approximation result with the original pulse.

This volume facilitates the use and calculation of spheroidal wave functions with a detailed and unified account of the properties of these functions and helpful tables.

"Tabulation of explicit values of radial spheroidal wave functions of both oblate and prolate kinds over extended ranges of parameters. It is designed to provide the mathematical physicist and research engineer with accurate values of important but not easily calculated functions needed to solve boundary value problems of radiation, scattering and propagation of scolar or vector waves in spheroidal coordinates."--Foreword.

The theory of prolate spheroidal wave functions is briefly reviewed. Formulas useful for the numerical calculation of prolate radial function of the first and second type, together with their first derivatives, are derived and explained. A step-by-step procedure of computation is then outlined together with an indication of precision achieved and method of checking. Finally a computer-printout tabulation of 66,600 entries comprising prolate radial functions and their first derivatives of both types is reproduced. The range of parameters m, l, h, and xi covered in these tables is m=0 and 1, l=m(1)18, h=0.1(0.1)1(0.2)8, and xi=1.01(0.01) 1.10. (Author).

"This beautiful book can be read as a novel presenting carefully our quest to get more and more information from our observations and measurements. Its authors are particularly good at relating it." --Pierre C. Sabatier "This is a unique text - a labor of love pulling together for the first time the remarkably large array of mathematical and statistical techniques used for analysis of resolution in many systems of importance today – optical, acoustical, radar, etc.... I believe it will find widespread use and value." --Dr. Robert G.W. Brown, Chief Executive Officer, American Institute of Physics "The mix of physics and mathematics is a unique feature of this book which can be basic not only for PhD students but also for researchers in the area of computational imaging." --Mario Bertero, Professor, University of Geneva "a tour-de-force covering aspects of history, mathematical theory and practical applications. The authors provide a penetrating insight into the often confused topic of resolution and in doing offer a unifying approach to the subject that is applicable not only to traditional optical systems but also modern day, computer-based systems such as radar and RF communications." --Prof. Ian Proudler, Loughborough University "a ‘must have’ for anyone interested in imaging and the spatial resolution of images. This book provides detailed and very readable account of resolution in imaging and organizes the recent history of the subject in excellent fashion.... I strongly recommend it." --Michael A. Fiddy, Professor, University of North Carolina at Charlotte This book brings together the concept of resolution, which limits what we can determine about our physical world, with the theory of linear inverse problems, emphasizing practical applications. The book focuses on methods for solving illposed problems that do not have unique stable solutions. After introducing basic concepts, the contents address problems with "continuous" data in detail before turning to cases of discrete data sets. As one of the unifying principles of the text, the authors explain how non-uniqueness is a feature of measurement problems in science where precision and resolution is essentially always limited by some kind of noise.

The volume is one of a series of six volumes published by the Naval Research Laboratory containing tabulation of explicit values of radial spheroidal wave functions of both oblate and prolate kinds over extended ranges of parameters. It is designed to provide the mathematical physicist and research engineer with accurate values of important but not easily calculated functions needed to solve boundary value problems of radiation, scattering, and propagation of scalar or vector waves in spheroidal coordinates. This series vastly extends the scope and accuracy of existing tabulations of radial spheroidal wave functions. The presence of many of the entries was made possible only through adoption of calculation techniques involving extreme precision. This was particularly true in the calculation of the characteristic values for the radial equation resulting from separation of the Helmholtz wave equation in spheroidal coordinates, a knowledge of which is essential in the calculation of spheroidal angle functions. The present document consists of Volume 1-prolate m=0. (Author).

The flagship monograph addressing the spheroidal wave function andits pertinence to computational electromagnetics Spheroidal Wave Functions in Electromagnetic Theorypresents in detail the theory of spheroidal wave functions, itsapplications to the analysis of electromagnetic fields in variousspheroidal structures, and provides comprehensive programming codesfor those computations. The topics covered in this monograph include: Spheroidal coordinates and wave functions Dyadic Green's functions in spheroidal systems EM scattering by a conducting spheroid EM scattering by a coated dielectric spheroid Spheroid antennas SAR distributions in a spheroidal head model The programming codes and their applications are provided onlineand are written in Mathematica 3.0 or 4.0. Readers can also developtheir own codes according to the theory or routine described in thebook to find subsequent solutions of complicated structures. Spheroidal Wave Functions in Electromagnetic Theory is afundamental reference for scientists, engineers, and graduatestudents practicing modern computational electromagnetics orapplied physics.

The Applied and Numerical Harmonic Analysis (ANHA) book series aims to provide the engineering, mathematical, and scienti?c communities with s- ni?cant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series re?ects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the int- leaving of theory and applications and their creative symbiotic evolution is axiomatic. Harmonic analysis is a wellspring of ideas and applicability that has ?o- ished, developed, and deepened over time within many disciplines and by means of creative cross-fertilizationwith diverse areas. The intricate and f- damental relationship between harmonic analysis and ?elds such as signal processing, partial di?erential equations (PDEs), and image processing is - ?ected in our state-of-the-art ANHA series. Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

Prolate Spheroidal Wave Functions (PSWFs) are the eigenfunctions of the bandlimited operator in one dimension. As such, they play an important role in signal processing, Fourier analysis, and approximation theory. While historically the numerical evaluation of PSWFs presented serious difficulties, the developments of the last fifteen years or so made them as computationally tractable as any other class of special functions. As a result, PSWFs have been becoming a popular computational tool. The present book serves as a complete, self-contained resource for both theory and computation. It will be of interest to a wide range of scientists and engineers, from mathematicians interested in PSWFs as an analytical tool to electrical engineers designing filters and antennas.

This volume contains articles based on talks presented at the Special Session Frames and Operator Theory in Analysis and Signal Processing, held in San Antonio, Texas, in January of 2006. Recently, the field of frames has undergone tremendous advancement. Most of the work in this field is focused on the design and construction of more versatile frames and frames tailored towards specific applications, e.g., finite dimensional uniform frames for cellular communication. In addition, frames are now becoming a hot topic in mathematical research as a part of many engineering applications, e.g., matching pursuits and greedy algorithms for image and signal processing. Topics covered in this book include: Application of several branches of analysis (e.g., PDEs; Fourier, wavelet, and harmonic analysis; transform techniques; data representations) to industrial and engineering problems, specifically image and signal processing. Theoretical and applied aspects of frames and wavelets. Pure aspects of operator theory emphasizing the connections to applied mathematics, frames, and signal processing. This volume will be equally attractive to pure mathematicians working on the foundations of frame and operator theory and their interconnections as it will to applied mathematicians investigating applications and to physicists and engineers employing these designs. It thus may appeal to a wide target group of researchers and may serve as a catalyst for cross-fertilization of several important areas of mathematics and the applied sciences.

Containing important new material unavailable previously in book form, this book covers a wide variety of topics which will be great interest to applied mathematicians and engineers. Introducing the main ideas, background material is provided on Fourier analysis, Hilbert spaces, and theirbases, before the book moves on to discuss more complex topics and their applications.

An extensive summary of mathematical functions that occur in physical and engineering problems

A method of calculating the eigenvalues and the expansion coefficients of spheroidal wave functions for the prolate and oblate case using a high-speed computer is presented. Special emphasis is placed on a new explanation of the role the parameter n plays in the numerical determination of eigenvalues. Curves and tables of eigenvalues are included.

If the electric field intensity in the Fraunhofer region of a one-dimensional radiating source can be represented as a finite Fourier transform of the source current, then the source current can be reconstructed exactly by using prolate spheroidal wave functions and a segment of either the far field or the diffraction-limited image for the noise-free case. An example of the image enhancement of this process is given for the case of two equal point sources, which are unresolved in the Rayleigh sense. The point response function of this process shows that the resolution cell extent can be readily reduced to less than 10% of the Rayleigh cell with only 20 degrees of enhancement processing. A method of generating the Legendre polynomial and power series expansions of the prolate spheroidal angle functions of the first kind and order zero was worked out in detail. The Legendre polynomial expansion coefficients for degrees n = 0(1)40 and the power series expansion coefficients for degrees n = 0(1)36 are tabulated for c = 2. (Author).

The solutions of the Helmholtz wave equation in prolate spheroidal coordinates can be obtained by separation of variables. The subject of this report is a Fortran computer program called PRAD which calculates numerical values to the solutions of the resulting ordinary differential equation for the 'radial' coordinate. The printed output of PRAD consists of radial functions of the first and second types, their first derivatives, the separation constants or eigenvalues, and an accuracy check. The report describes the computer program PRAD and briefly reviews the theory of prolate spheroidal wave functions. A computer listing of PRAD along with some sample output is included in an appendix. (Author).

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