Matrix Lie Groups And Lie Groups Michigan State University

Introduction to Lie Groups and Lie Algebra, 51

Matrix groups touch an enormous spectrum of the mathematical arena. This textbook brings them into the undergraduate curriculum. It makes an excellent one-semester course for students familiar with linear and abstract algebra and prepares them for a graduate course on Lie groups. Matrix Groups for Undergraduates is concrete and example-driven, with geometric motivation and rigorous proofs. The story begins and ends with the rotations of a globe. In between, the author combines rigor and intuition to describe the basic objects of Lie theory: Lie algebras, matrix exponentiation, Lie brackets, maximal tori, homogeneous spaces, and roots. This second edition includes two new chapters that allow for an easier transition to the general theory of Lie groups.

This introduction to the theory of lie groups and their representations starts from basic undergraduate maths and proceeds through the fundamentals of Lie theory to topics in representation theory, such as the Peter-Weyl theorem.

This book is intended to serve as a textbook for a one-semester course for M.Sc/M.Phil. Students at Indian universities. Students of theoretical physics will also find this exposition useful. The general theory of Lie groups appears formidable to an M.Sc./M.Phil. student.

This textbook offers an introduction to differential geometry designed for readers interested in modern geometry processing. Working from basic undergraduate prerequisites, the authors develop manifold theory and Lie groups from scratch; fundamental topics in Riemannian geometry follow, culminating in the theory that underpins manifold optimization techniques. Students and professionals working in computer vision, robotics, and machine learning will appreciate this pathway into the mathematical concepts behind many modern applications. Starting with the matrix exponential, the text begins with an introduction to Lie groups and group actions. Manifolds, tangent spaces, and cotangent spaces follow; a chapter on the construction of manifolds from gluing data is particularly relevant to the reconstruction of surfaces from 3D meshes. Vector fields and basic point-set topology bridge into the second part of the book, which focuses on Riemannian geometry. Chapters on Riemannian manifolds encompass Riemannian metrics, geodesics, and curvature. Topics that follow include submersions, curvature on Lie groups, and the Log-Euclidean framework. The final chapter highlights naturally reductive homogeneous manifolds and symmetric spaces, revealing the machinery needed to generalize important optimization techniques to Riemannian manifolds. Exercises are included throughout, along with optional sections that delve into more theoretical topics. Differential Geometry and Lie Groups: A Computational Perspective offers a uniquely accessible perspective on differential geometry for those interested in the theory behind modern computing applications. Equally suited to classroom use or independent study, the text will appeal to students and professionals alike; only a background in calculus and linear algebra is assumed. Readers looking to continue on to more advanced topics will appreciate the authors’ companion volume Differential Geometry and Lie Groups: A Second Course.

This volume treats algebraic groups from a group theoretical point of view and compares the results with the analogous issues in the theory of Lie groups. It examines a classification of algebraic groups and Lie groups having only few subgroups.

Symmetry Groups and Their Applications

Lie Groups Beyond an Introduction takes the reader from the end of introductory Lie group theory to the threshold of infinite-dimensional group representations. Merging algebra and analysis throughout, the author uses Lie-theoretic methods to develop a beautiful theory having wide applications in mathematics and physics. A feature of the presentation is that it encourages the reader's comprehension of Lie group theory to evolve from beginner to expert: initial insights make use of actual matrices, while later insights come from such structural features as properties of root systems, or relationships among subgroups, or patterns among different subgroups.

This book has grown out of a set of lecture notes I had prepared for a course on Lie groups in 1966. When I lectured again on the subject in 1972, I revised the notes substantially. It is the revised version that is now appearing in book form. The theory of Lie groups plays a fundamental role in many areas of mathematics. There are a number of books on the subject currently available -most notably those of Chevalley, Jacobson, and Bourbaki-which present various aspects of the theory in great depth. However, 1 feei there is a need for a single book in English which develops both the algebraic and analytic aspects of the theory and which goes into the representation theory of semi simple Lie groups and Lie algebras in detail. This book is an attempt to fiii this need. It is my hope that this book will introduce the aspiring graduate student as well as the nonspecialist mathematician to the fundamental themes of the subject. I have made no attempt to discuss infinite-dimensional representations. This is a very active field, and a proper treatment of it would require another volume (if not more) of this size. However, the reader who wants to take up this theory will find that this book prepares him reasonably well for that task.

This book is intended for a one-year graduate course on Lie groups and Lie algebras. The book goes beyond the representation theory of compact Lie groups, which is the basis of many texts, and provides a carefully chosen range of material to give the student the bigger picture. The book is organized to allow different paths through the material depending on one's interests. This second edition has substantial new material, including improved discussions of underlying principles, streamlining of some proofs, and many results and topics that were not in the first edition. For compact Lie groups, the book covers the Peter–Weyl theorem, Lie algebra, conjugacy of maximal tori, the Weyl group, roots and weights, Weyl character formula, the fundamental group and more. The book continues with the study of complex analytic groups and general noncompact Lie groups, covering the Bruhat decomposition, Coxeter groups, flag varieties, symmetric spaces, Satake diagrams, embeddings of Lie groups and spin. Other topics that are treated are symmetric function theory, the representation theory of the symmetric group, Frobenius–Schur duality and GL(n) × GL(m) duality with many applications including some in random matrix theory, branching rules, Toeplitz determinants, combinatorics of tableaux, Gelfand pairs, Hecke algebras, the "philosophy of cusp forms" and the cohomology of Grassmannians. An appendix introduces the reader to the use of Sage mathematical software for Lie group computations.

This self-contained text is an excellent introduction to Lie groups and their actions on manifolds. The authors start with an elementary discussion of matrix groups, followed by chapters devoted to the basic structure and representation theory of finite dimensinal Lie algebras. They then turn to global issues, demonstrating the key issue of the interplay between differential geometry and Lie theory. Special emphasis is placed on homogeneous spaces and invariant geometric structures. The last section of the book is dedicated to the structure theory of Lie groups. Particularly, they focus on maximal compact subgroups, dense subgroups, complex structures, and linearity. This text is accessible to a broad range of mathematicians and graduate students; it will be useful both as a graduate textbook and as a research reference.

This text introduces upper-level undergraduates to Lie group theory and physical applications. It further illustrates Lie group theory's role in several fields of physics. 1974 edition. Includes 75 figures and 17 tables, exercises and problems.

This textbook treats Lie groups, Lie algebras and their representations in an elementary but fully rigorous fashion requiring minimal prerequisites. In particular, the theory of matrix Lie groups and their Lie algebras is developed using only linear algebra, and more motivation and intuition for proofs is provided than in most classic texts on the subject. In addition to its accessible treatment of the basic theory of Lie groups and Lie algebras, the book is also noteworthy for including: a treatment of the Baker–Campbell–Hausdorff formula and its use in place of the Frobenius theorem to establish deeper results about the relationship between Lie groups and Lie algebras motivation for the machinery of roots, weights and the Weyl group via a concrete and detailed exposition of the representation theory of sl(3;C) an unconventional definition of semisimplicity that allows for a rapid development of the structure theory of semisimple Lie algebras a self-contained construction of the representations of compact groups, independent of Lie-algebraic arguments The second edition of Lie Groups, Lie Algebras, and Representations contains many substantial improvements and additions, among them: an entirely new part devoted to the structure and representation theory of compact Lie groups; a complete derivation of the main properties of root systems; the construction of finite-dimensional representations of semisimple Lie algebras has been elaborated; a treatment of universal enveloping algebras, including a proof of the Poincaré–Birkhoff–Witt theorem and the existence of Verma modules; complete proofs of the Weyl character formula, the Weyl dimension formula and the Kostant multiplicity formula. Review of the first edition: This is an excellent book. It deserves to, and undoubtedly will, become the standard text for early graduate courses in Lie group theory ... an important addition to the textbook literature ... it is highly recommended. — The Mathematical Gazette

Introduces the concepts and methods of the Lie theory in a form accessible to the nonspecialist by keeping mathematical prerequisites to a minimum. Although the authors have concentrated on presenting results while omitting most of the proofs, they have compensated for these omissions by including many references to the original literature. Their treatment is directed toward the reader seeking a broad view of the subject rather than elaborate information about technical details. Illustrations of various points of the Lie theory itself are found throughout the book in material on applications. In this reprint edition, the authors have resisted the temptation of including additional topics. Except for correcting a few minor misprints, the character of the book, especially its focus on classical representation theory and its computational aspects, has not been changed.

Lie algebras are efficient tools for analyzing the properties of physical systems. Concrete applications comprise the formulation of symmetries of Hamiltonian systems, the description of atomic, molecular and nuclear spectra, the physics of elementary particles and many others. This work gives an introduction to the properties and the structure of the Lie algebras su(n). The book features an elementary (matrix) access to su(N)-algebras, and gives a first insight into Lie algebras. Student readers should be enabled to begin studies on physical su(N)-applications, instructors will profit from the detailed calculations and examples.

The book presents the main approaches in study of algebraic structures of symmetries in models of theoretical and mathematical physics, namely groups and Lie algebras and their deformations. It covers the commonly encountered quantum groups (including Yangians). The second main goal of the book is to present a differential geometry of coset spaces that is actively used in investigations of models of quantum field theory, gravity and statistical physics. The third goal is to explain the main ideas about the theory of conformal symmetries, which is the basis of the AdS/CFT correspondence. The theory of groups and symmetries is an important part of theoretical physics. In elementary particle physics, cosmology and related fields, the key role is played by Lie groups and algebras corresponding to continuous symmetries. For example, relativistic physics is based on the Lorentz and Poincare groups, and the modern theory of elementary particles — the Standard Model — is based on gauge (local) symmetry with the gauge group SU(3) x SU(2) x U(1). This book presents constructions and results of a general nature, along with numerous concrete examples that have direct applications in modern theoretical and mathematical physics. Contents: Preface Groups and Transformations Lie Groups Lie Algebras Representations of Groups and Lie Algebras Compact Lie Algebras Root Systems and Classification of Simple Lie Algebras Homogeneous Spaces and their Geometry Solutions to Selected Problems Selected Bibliography References Index Readership: Graduate students and researchers in theoretical physics and mathematical physics. Keywords: Lie Groups;Lie Algebras;Representation Theory;Conformal Symmetries;Yangians;Coset Spaces;Differential Geometry;Casimir Operators;Root Systems;AdS Spaces;Lobachevskian GeometryReview:0

Eight papers provide mature readers with careful review of progress (through about 1983) toward the creation of a theory of the representations of infinite-dimensional Lie groups and algebras, and of some related topics. Recent developments in physics have provided major impetus for the development of such a theory, and the volume will be of special interest to mathematical physicists (quantum field theorists). Translated from the Russian edition of unstated date, and beautifully produced (which--at the price--it should be!). Book club price, $118. (NW) Annotation copyrighted by Book News, Inc., Portland, OR

The subject of Lie groups is one that slips by many a mathematician. Many claim that the topic is not accessible to undergraduate research. The book Lie Groups by Harriet Pollatsek came out a few years ago, and it was meant to be a new way to be introduced to the topic. However, the book does not quite get far enough to give a formal definition of a Lie group. The goal of this project is to "bridge the gap." The objective of this thesis is to include all the introductory material required to get to where the definition of a Lie group is no longer something so complicated. We will illustrate the major concepts by examples. Many matrix groups are Lie groups. Matrix groups are well-known, and they are an ideal place to start learning about what a Lie group can do. We then look at tangent spaces of the matrix groups, or the Lie algebra that is associated with each Lie group. After some motivation behind Lie algebras, we finally get to the feature presentation: a group and a differentiable manifold, put together into one super structure known as a Lie group.

This volume begins with an introduction to the structure of finite-dimensional simple Lie algebras, including the representation of ${\widehat {\mathfrak {sl}}}(2, {\mathbb C})$, root systems, the Cartan matrix, and a Dynkin diagram of a finite-dimensional simple Lie algebra. Continuing on, the main subjects of the book are the structure (real and imaginary root systems) of and the character formula for Kac-Moody superalgebras, which is explained in a very general setting. Only elementary linear algebra and group theory are assumed. Also covered is modular property and asymptotic behavior of integrable characters of affine Lie algebras. The exposition is self-contained and includes examples. The book can be used in a graduate-level course on the topic.

This book reproduces J-P. Serre's 1964 Harvard lectures. The aim is to introduce the reader to the "Lie dictionary": Lie algebras and Lie groups. Special features of the presentation are its emphasis on formal groups (in the Lie group part) and the use of analytic manifolds on p-adic fields. Some knowledge of algebra and calculus is required of the reader, but the text is easily accessible to graduate students, and to mathematicians at large.

This book offers a first taste of the theory of Lie groups, focusing mainly on matrix groups: closed subgroups of real and complex general linear groups. The first part studies examples and describes classical families of simply connected compact groups. The second section introduces the idea of a lie group and explores the associated notion of a homogeneous space using orbits of smooth actions. The emphasis throughout is on accessibility.

There are two approaches to compact lie groups: by computation as matrices or theoretically as manifolds with a group structure. The great appeal of this book is the blending of these two approaches. The theoretical results are illustrated by computations and the theory provides a commentary on the computational work. Indeed, there are extensive computations of the structure and representation theory for the classical groups SU(n), SO(n) and Sp(n). A second exciting feature is that the differential geometry of a compact Lie group, both the classical curvature studies and the more recent heat equation methods, are treated. A large number of formulas for the connection and curvature are conveniently gathered together. This book provides an excellent text for a first course in compact Lie groups. Request Inspection Copy

The book presents examples of important techniques and theorems for Groups, Lie groups and Lie algebras. This allows the reader to gain understandings and insights through practice. Applications of these topics in physics and engineering are also provided. The book is self-contained. Each chapter gives an introduction to the topic.

Introduction to Lie groups and Lie algebras

Presents an innovative synthesis of methods used to study problems of equivalence and symmetry.

The book is intended for graduate students of theoretical physics (with a background in quantum mechanics) as well as researchers interested in applications of Lie group theory and Lie algebras in physics. The emphasis is on the inter-relations of representation theories of Lie groups and the corresponding Lie algebras.

There exist two correspondences between groups and Lie algebras. One occurs between matrix Lie groups and Lie algebras. The other concerns itself with complete groups of unitriangular matrices and Lie algebras. Titled the Lie and Mal'cev correspondences respectively, the purpose of this paper is to explore the two. We begin with an introduction to the basic properties of Lie algebras and other preliminary material followed by a construction of free Lie rings and algebras as well as by other interesting material discovered along the way. We then dive into the Lie correspondence, with which we contrast the Mal'cev correspondence in the section after.

This textbook is a complete introduction to Lie groups for undergraduate students. The only prerequisites are multi-variable calculus and linear algebra. The emphasis is placed on the algebraic ideas, with just enough analysis to define the tangent space and the differential and to make sense of the exponential map. This textbook works on the principle that students learn best when they are actively engaged. To this end nearly 200 problems are included in the text, ranging from the routine to the challenging level. Every chapter has a section called 'Putting the pieces together' in which all definitions and results are collected for reference and further reading is suggested.

Polished lecture notes provide a clean and usefully detailed account of the standard elements of the theory of Lie groups and algebras. Following nineteen pages of preparatory material, Part I (seven brief chapters) treats "Lie groups and their Lie algebras"; Part II (seven chapters) treats "complex semi-simple Lie algebras"; Part III (two chapters) treats "real semi-simple Lie algebras". The page design is intimidatingly dense, the exposition very much in the familiar "definition/lemma/proof/theorem/proof/remark" mode, and there are no exercises or bibliography. (NW) Annotation copyrighted by Book News, Inc., Portland, OR

Matrix Inequalities and Their Extensions to Lie Groups gives a systematic and updated account of recent important extensions of classical matrix results, especially matrix inequalities, in the context of Lie groups. It is the first systematic work in the area and will appeal to linear algebraists and Lie group researchers.

This book addresses Lie groups, Lie algebras, and representation theory. The author restricts attention to matrix Lie groups and Lie algebras. This approach keeps the discussion concrete, allows the reader to get to the heart of the subject quickly, and covers all of the most interesting examples.From the reviews:"Sure to become a standard textbook for graduate students in mathematics and physics with little or no prior exposure to Lie theory." --L'Enseignement Mathematique

Complex Lie groups have often been used as auxiliaries in the study of real Lie groups in areas such as differential geometry and representation theory. To date, however, no book has fully explored and developed their structural aspects. The Structure of Complex Lie Groups addresses this need. Self-contained, it begins with general concepts introduced via an almost complex structure on a real Lie group. It then moves to the theory of representative functions of Lie groups- used as a primary tool in subsequent chapters-and discusses the extension problem of representations that is essential for studying the structure of complex Lie groups. This is followed by a discourse on complex analytic groups that carry the structure of affine algebraic groups compatible with their analytic group structure. The author then uses the results of his earlier discussions to determine the observability of subgroups of complex Lie groups. The differences between complex algebraic groups and complex Lie groups are sometimes subtle and it can be difficult to know which aspects of algebraic group theory apply and which must be modified. The Structure of Complex Lie Groups helps clarify those distinctions. Clearly written and well organized, this unique work presents material not found in other books on Lie groups and serves as an outstanding complement to them.

This book starts with the elementary theory of Lie groups of matrices and arrives at the definition, elementary properties, and first applications of cohomological induction, which is a recently discovered algebraic construction of group representations. Along the way it develops the computational techniques that are so important in handling Lie groups. The book is based on a one-semester course given at the State University of New York, Stony Brook in fall, 1986 to an audience having little or no background in Lie groups but interested in seeing connections among algebra, geometry, and Lie theory. These notes develop what is needed beyond a first graduate course in algebra in order to appreciate cohomological induction and to see its first consequences. Along the way one is able to study homological algebra with a significant application in mind; consequently one sees just what results in that subject are fundamental and what results are minor.

This two-volume set covers stochastic processes, information theory and Lie groups in a unified setting, bridging topics rarely studied together. The emphasis is on using stochastic, geometric, and group-theoretic concepts for modeling physical phenomena.

Matrix GroupsAn Introduction to Lie Group TheorySpringer Science & Business Media

Lie groups and Lie algebras have become essential to many parts of mathematics and theoretical physics, with Lie algebras a central object of interest in their own right. This book provides an elementary introduction to Lie algebras based on a lecture course given to fourth-year undergraduates. The only prerequisite is some linear algebra and an appendix summarizes the main facts that are needed. The treatment is kept as simple as possible with no attempt at full generality. Numerous worked examples and exercises are provided to test understanding, along with more demanding problems, several of which have solutions. Introduction to Lie Algebras covers the core material required for almost all other work in Lie theory and provides a self-study guide suitable for undergraduate students in their final year and graduate students and researchers in mathematics and theoretical physics.

From the reviews of the French edition: "This is a rich and useful volume. The material it treats has relevance well beyond the theory of Lie groups and algebras, ranging from the geometry of regular polytopes and paving problems to current work on finite simple groups having a (B,N)-pair structure, or ‘Tits systems’". --G.B. Seligman in MathReviews.

In 1991-1993 our three-volume book "Representation of Lie Groups and Spe cial Functions" was published. When we started to write that book (in 1983), editors of "Kluwer Academic Publishers" expressed their wish for the book to be of encyclopaedic type on the subject. Interrelations between representations of Lie groups and special functions are very wide. This width can be explained by existence of different types of Lie groups and by richness of the theory of their rep resentations. This is why the book, mentioned above, spread to three big volumes. Influence of representations of Lie groups and Lie algebras upon the theory of special functions is lasting. This theory is developing further and methods of the representation theory are of great importance in this development. When the book "Representation of Lie Groups and Special Functions" ,vol. 1-3, was under preparation, new directions of the theory of special functions, connected with group representations, appeared. New important results were discovered in the traditional directions. This impelled us to write a continuation of our three-volume book on relationship between representations and special functions. The result of our further work is the present book. The three-volume book, published before, was devoted mainly to studying classical special functions and orthogonal polynomials by means of matrix elements, Clebsch-Gordan and Racah coefficients of group representations and to generaliza tions of classical special functions that were dictated by matrix elements of repre sentations.

Lie groups has been an increasing area of focus and rich research since the middle of the 20th century. In Lie Groups: An Approach through Invariants and Representations, the author's masterful approach gives the reader a comprehensive treatment of the classical Lie groups along with an extensive introduction to a wide range of topics associated with Lie groups: symmetric functions, theory of algebraic forms, Lie algebras, tensor algebra and symmetry, semisimple Lie algebras, algebraic groups, group representations, invariants, Hilbert theory, and binary forms with fields ranging from pure algebra to functional analysis. By covering sufficient background material, the book is made accessible to a reader with a relatively modest mathematical background. Historical information, examples, exercises are all woven into the text. This unique exposition is suitable for a broad audience, including advanced undergraduates, graduates, mathematicians in a variety of areas from pure algebra to functional analysis and mathematical physics.